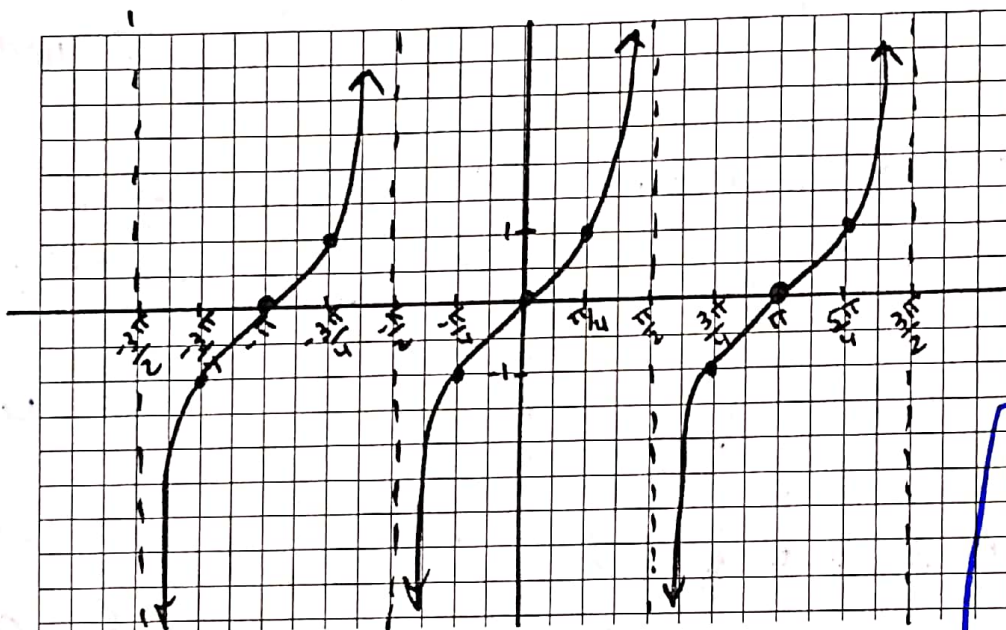## 4.5 Graphs of Other Trigonometric Functions

Evaluate $y = \tan\theta$ for the multiples of $\dfrac{\pi}{4}$ in the interval $\dfrac{-3\pi}{2} \leq \theta \leq \dfrac{3\pi}{2}$

θ	$-\dfrac{3\pi}{2}$	$-\dfrac{5\pi}{4}$	$-\pi$	$-\dfrac{3\pi}{4}$	$-\dfrac{\pi}{2}$	$-\dfrac{\pi}{4}$	0	$\dfrac{\pi}{4}$	$\dfrac{\pi}{2}$	$\dfrac{3\pi}{4}$	π	$\dfrac{5\pi}{4}$	$\dfrac{3\pi}{2}$
Tan θ $\dfrac{y}{x}$	undef.	-1	0	1	undef.	-1	0	1	undef	-1	0	1	undef

one per.
tan θ

Graph $y = \tan\theta$

one period
of tan θ

one period
of tan θ
V. A to V. A
$X = -\dfrac{\pi}{2}$ to $X = \pi/2$
V.A. -1 0 1 V.A

(5 critical
values are
V. A.s and
points)

critical values
for one period.
V.A s $X = -\dfrac{\pi}{2}$
$X = \dfrac{\pi}{2}$
pts $(-\dfrac{\pi}{4}, -1)$
$(0, 0)$
$(\dfrac{\pi}{4}, 1)$

Properties of the graph of $y = \tan\theta$

1. The period is ___ π ___.
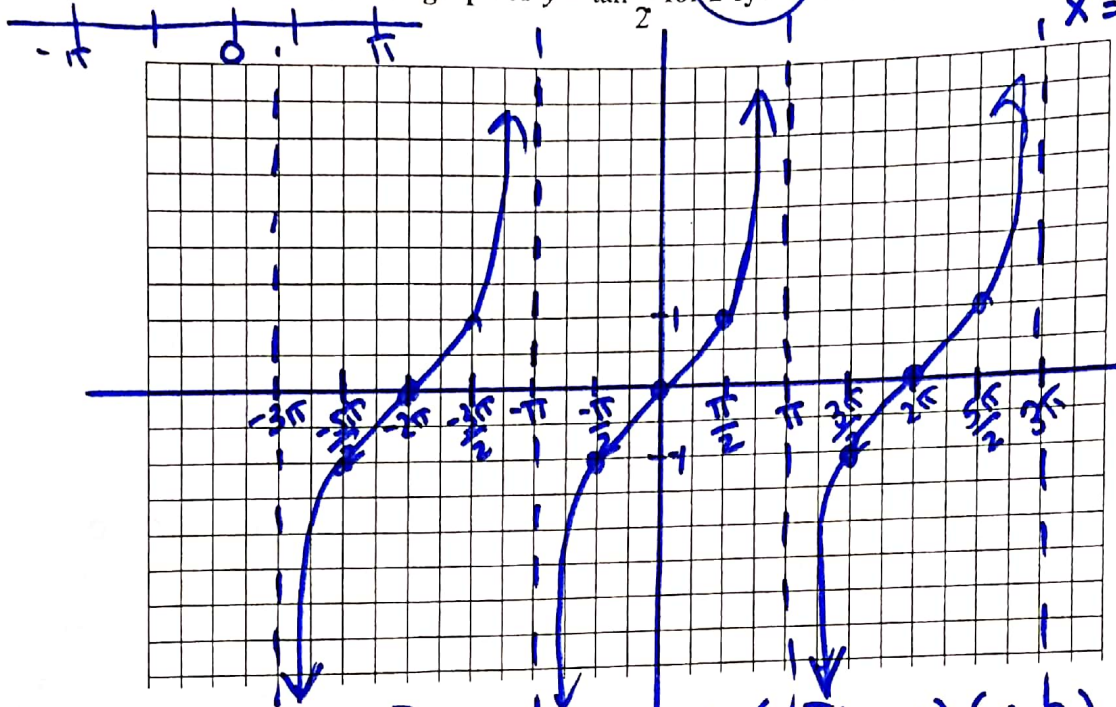
2. The domain is the set of real numbers except $\dfrac{\pi}{2}n$, where n is an odd integer.

3. The range is the set of real numbers. $(-\infty, \infty)$

4. The x intercepts are located at πn, where n is an integer.

5. The y intercept is (0, 0).

6. The vertical asymptotes are $x = \dfrac{\pi}{2}n$, where n is an odd integer.

$$Per = \frac{\pi}{1/2} = \underline{2\pi} \qquad y = \tan\left(\frac{1}{2}x\right) \xleftarrow{\text{H. stretch by 2}}$$

Ex: 1 Sketch the graph of $y = \tan\frac{x}{2}$ for (2 cycles) of output values.

L. VA
$$\frac{1}{2}x = -\frac{\pi}{2}$$
$$x = -\pi$$

R. VA
$$\frac{1}{2}x = \frac{\pi}{2}$$
$$x = \pi$$

x	y	VA
$-\pi$	undef	$x = -\pi$
$-\frac{\pi}{2}$	-1	
0	0	
$\frac{\pi}{2}$	1	
π	undef	$x = \pi$

V. AS $x = -\pi, \ x = \pi$ points $\left(\frac{-\pi}{2}, -1\right) \ (0, 0) \ \left(\frac{\pi}{2}, 1\right)$

Ex: 2 Sketch the graph of $y = -3\tan(2x)$ for 2 cycles of output values.

$$Per = \frac{\pi}{2}$$

$$2x = -\frac{\pi}{2} \cdot \frac{1}{2} \qquad 2x = +\frac{\pi}{2} \cdot \frac{1}{2}$$
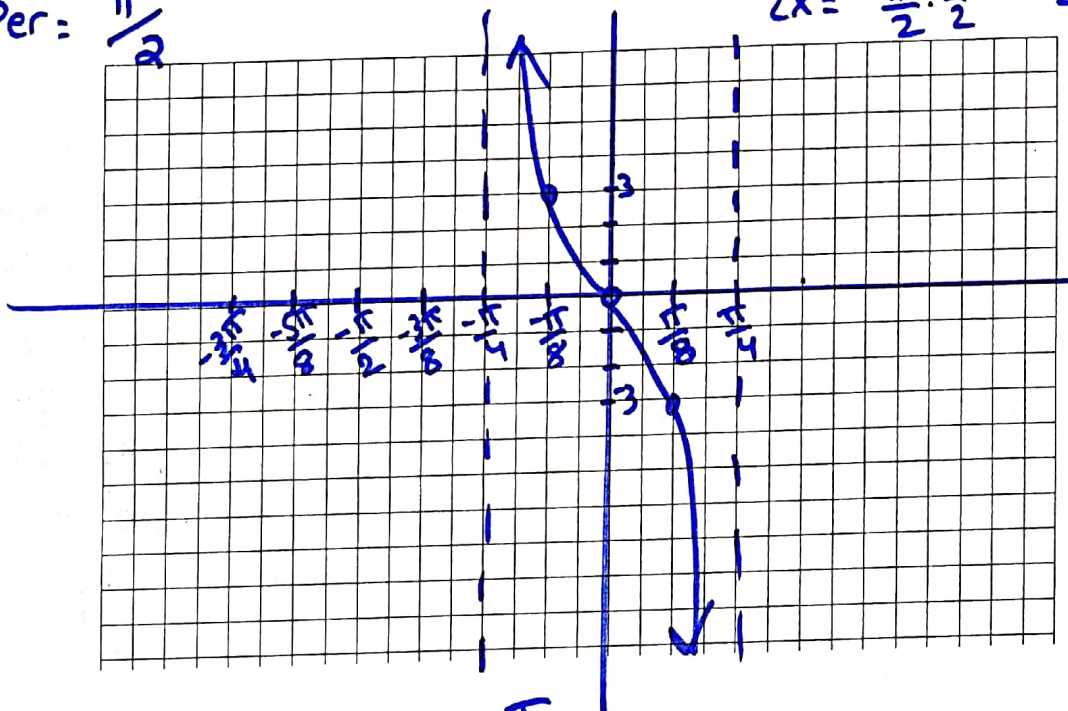
x	y
$-\frac{\pi}{4}$	und
$-\frac{\pi}{8}$	$-1(-3) = 3$
0	$0(-3) = 0$
$\frac{\pi}{8}$	$1(-3) = -3$
$\frac{\pi}{4}$	und

VA $\quad x = -\frac{\pi}{4}$

$\qquad x = \frac{\pi}{4}$

pts

$\left(-\frac{\pi}{8}, 3\right)$

$(0, 0)$

$\left(\frac{\pi}{8}, -3\right)$

1   0   -1

$x = \pi$ $x = 3\pi$

Per = $\frac{\pi}{1/3}$ = 3π

Ex: 3 Sketch the graph of $y = 2\cot\left(\dfrac{x}{3}\right)$ for 2 cycles of output values.

$y = 2\cot\left(\tfrac{1}{3}x\right)$

$\dfrac{x}{3} = 0$ $x = 0$ $\dfrac{x}{3} = \pi$

x	y
0	undef
$\dfrac{3\pi}{4}$	$1(2) = 2$
$\dfrac{3\pi}{2}$	$0(2) = 0$
$\dfrac{9\pi}{4}$	$-1(2) = -2$
3π	undef

Graph $y = \csc\theta$

$y = \dfrac{1}{\sin\theta}$

use pencil to draw $\sin\theta$

$\sin\theta \longrightarrow \csc\theta$

intersect $\rightarrow$ V.A.

incr $\longrightarrow$ decr

decr $\longrightarrow$ incr

$1 \rightarrow 1$

$-1 \rightarrow -1$

erase $\sin\theta$

Properties of the graph of $y = \csc\theta$.

1. The period is ___2π___.   2. The domain is the set of real numbers except πn, where n is an integer.

3. The range is $(-\infty, -1] \cup [1, \infty)$.   4. There is no x intercept.

5. There is no y intercept.   6. The vertical asymptotes are $x = \pi n$, where n is an integer.

7. y = 1 when $x = \dfrac{\pi}{2} + 2\pi n$, where n is an integer.   8. y = -1 when $x = \dfrac{3\pi}{2} + 2\pi n$, where n is an integer.